

Lattice Boltzmann CFD for Fusion Applications

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Main Topics

- Objectives
- Lattice Boltzmann method
- Application to MHD flows
- Variable grid spacing
- Turbulence simulation
- Conclusions & future work

Phase II Objectives

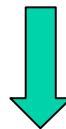
Development of Computational Techniques & Models

MHD Turbulence

Chemical reactions

Flow though complex geometry

Free-surface flow



Assemble parallel code
Pre- and post processors

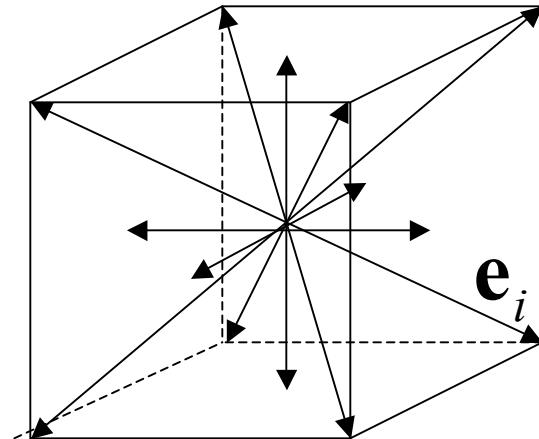


Fusion Application Package

Lattice Boltzmann Method

Solve for velocity distribution

$$\rho(\mathbf{x}) = \sum_i f_i(\mathbf{x}) \quad u_x(\mathbf{x}) = \sum_i e_{ix} f_i(\mathbf{x})$$



$$f_i(\mathbf{x} + \mathbf{e}_i, t+1) = f_i(\mathbf{x}, t) + \frac{f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)}{\tau}$$

$$f_i^{eq} = w_i \rho \left[1 + 3\mathbf{e}_i \cdot (\mathbf{u} + \tau \mathbf{a}) + \frac{9}{2} (\mathbf{e}_i \cdot (\mathbf{u} + \tau \mathbf{a}))^2 - \frac{3}{2} (\mathbf{u} + \tau \mathbf{a}) \cdot (\mathbf{u} + \tau \mathbf{a}) \right]$$

\mathbf{a} is force term

τ is a relaxation time (function of viscosity)

Magnetohydrodynamic (MHD) Equations

Fluid dynamical equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \nabla p + \mathbf{F}_{Lorentz} + \nabla \cdot (2\nu \rho \mathbf{S})$$

$$\mathbf{F}_{Lorentz} = \mathbf{J} \times \mathbf{B} = \nabla \cdot \left[-\frac{1}{2\mu} \mathbf{B} \cdot \mathbf{B} \mathbf{I} + \frac{1}{\mu} \mathbf{B} \cdot \nabla \mathbf{B} \right]$$

Maxwell stress tensor

Magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{u} \mathbf{B} - \mathbf{B} \mathbf{u}) = \eta \nabla^2 \mathbf{B} \quad \nabla \cdot \mathbf{B} = 0$$

$$\eta = \frac{1}{\sigma \mu}$$

Magnetic resistivity or
diffusivity

Lattice Kinetic Model for MHD – I

(Dellar, J. Comp. Phys. (2002); Breyiannis & Valougeorgis, Phys. Rev. E (2004))

Scalar distribution function for hydrodynamics

$$f_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta_t, t + \delta_t) - f_\alpha(\mathbf{x}, t) = -\frac{1}{\tau_{visc}}(f_\alpha - f_\alpha^{eq})$$

Vector distribution function for magnetic induction

$$\mathbf{g}_\beta(\mathbf{x} + \boldsymbol{\Xi}_\beta \delta_t, t + \delta_t) - \mathbf{g}_\beta(\mathbf{x}, t) = -\frac{1}{\tau_{mag}}(\mathbf{g}_\beta - \mathbf{g}_\beta^{eq})$$

Equilibrium distribution functions

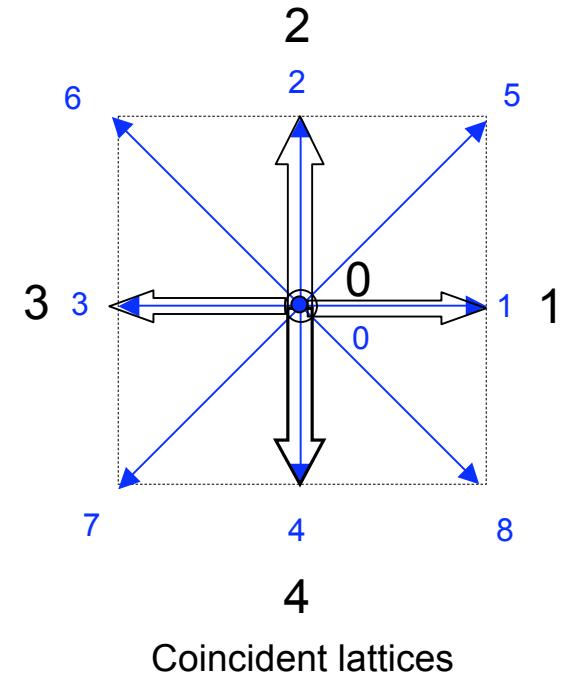
$$f_\alpha^{eq} = f_\alpha^{eq}(\rho, \mathbf{u}, \mathbf{B}) \quad \mathbf{g}_\alpha^{eq} = \mathbf{g}_\alpha^{eq}(\mathbf{u}, \mathbf{B})$$

Macroscopic fields

$$\rho = \sum_{\alpha=0}^8 f_\alpha \quad \rho \mathbf{u} = \sum_{\alpha=0}^8 f_\alpha \mathbf{e}_\alpha \quad \mathbf{B} = \sum_{\beta=0}^4 \mathbf{g}_\beta$$

Transport coefficients (Diffusivities)

$$\nu = \frac{c^2}{3} \left(\tau_{visc} - \frac{1}{2} \right) \delta_t \quad \eta = \frac{c^2}{3} \left(\tau_{mag} - \frac{1}{2} \right) \delta_t$$



$$\mathbf{e}_\alpha, \alpha = 0, 1, \dots, 8$$

$$\boldsymbol{\Xi}_\beta, \beta = 0, 1, \dots, 4$$

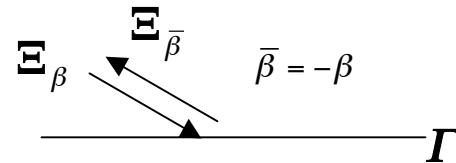
$$c = \frac{\delta_x}{\delta_t}$$

Lattice Kinetic Model for MHD – II

Boundary conditions

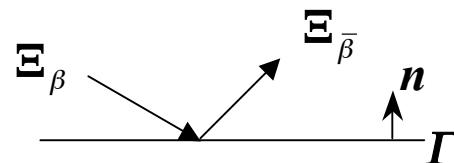
Bounce-back

$$\mathbf{B}(\Gamma) = \mathbf{0}$$



Specular reflection

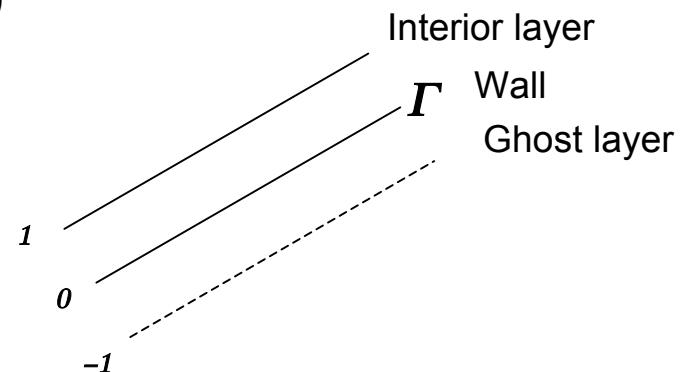
$$\frac{\partial \mathbf{B}}{\partial \mathbf{n}}(\Gamma) = \mathbf{0}$$



Extrapolation method (for more general
Dirichlet / Neumann boundary conditions)

$$\mathbf{g}_\alpha^{eq} \Big|_\Gamma = \mathbf{g}_\alpha^{eq} (\mathbf{u}_\Gamma, \mathbf{B}_\Gamma)$$

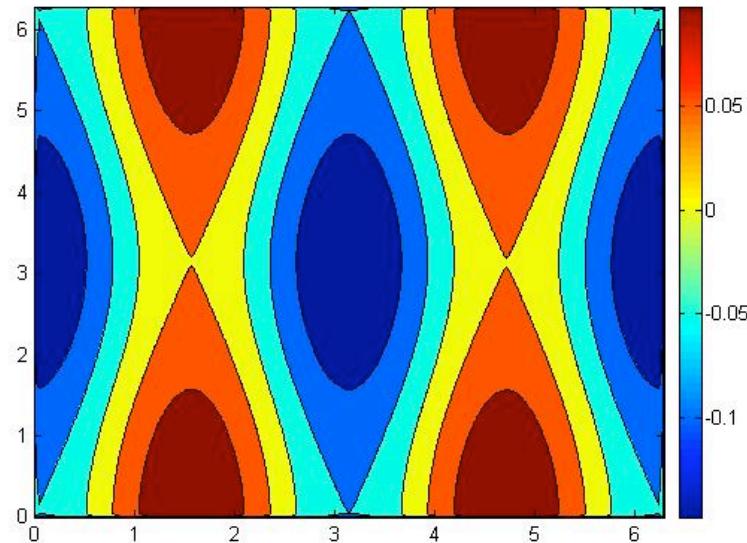
Post-collision $\mathbf{g}_\beta^{-1} = 2\mathbf{g}_\beta^0 - \mathbf{g}_\beta^1$



MHD Results - I

Orszag -Tang vortex

$t=0$



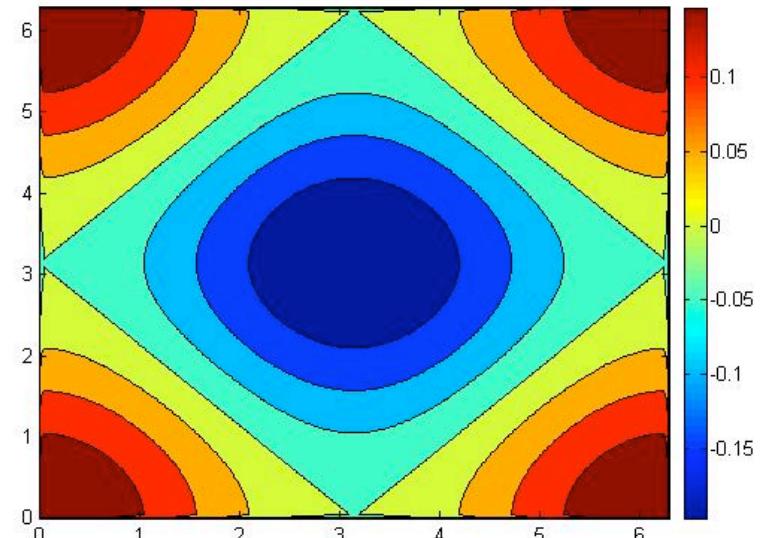
$$\mathbf{J} = j \hat{\mathbf{k}} = \nabla \times \mathbf{B}$$

Current density

Initial conditions

$$u_x(x, y, 0) = u_0 \sin y \quad B_x(x, y, 0) = -b_0 \sin y$$

$$u_y(x, y, 0) = u_0 \sin x \quad B_y(x, y, 0) = -b_0 \sin(2x)$$



$$\boldsymbol{\omega} = \boldsymbol{\omega} \hat{\mathbf{k}} = \nabla \times \mathbf{u}$$

Vorticity
Parameters

Domain size $2\pi \times 2\pi$

Resolution 256×256

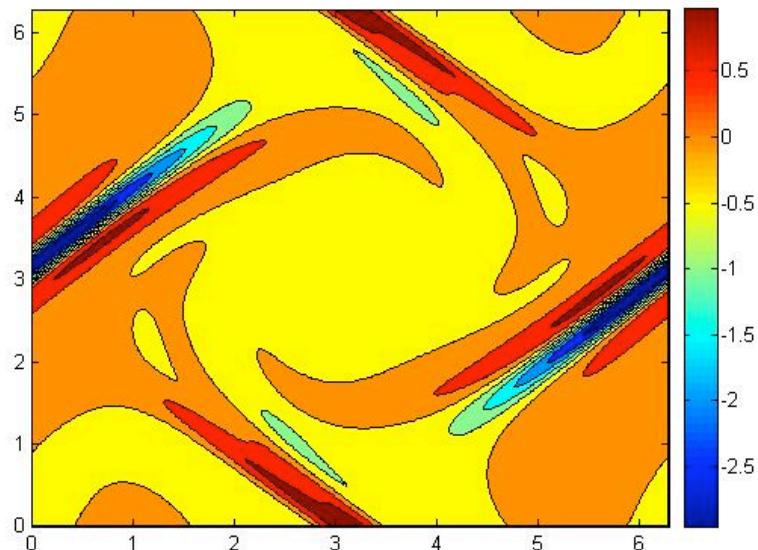
Reynolds numbers $\text{Re} = \text{Re}_m = 765$

$u_0 = 0.1, b_0 = 0.05, \tau_{visc} = \tau_{mag} = 0.60$

MHD Results - II

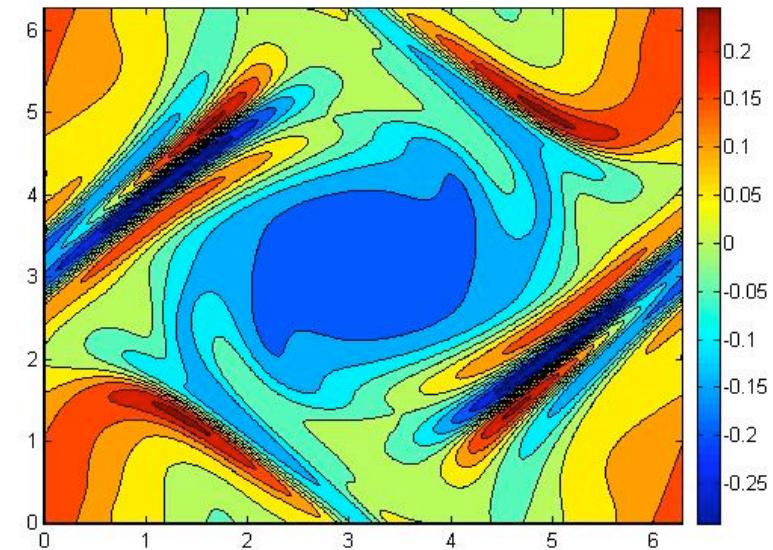
Orszag -Tang vortex

t=1200



$$\mathbf{J} = j\hat{\mathbf{k}} = \nabla \times \mathbf{B}$$

Current density



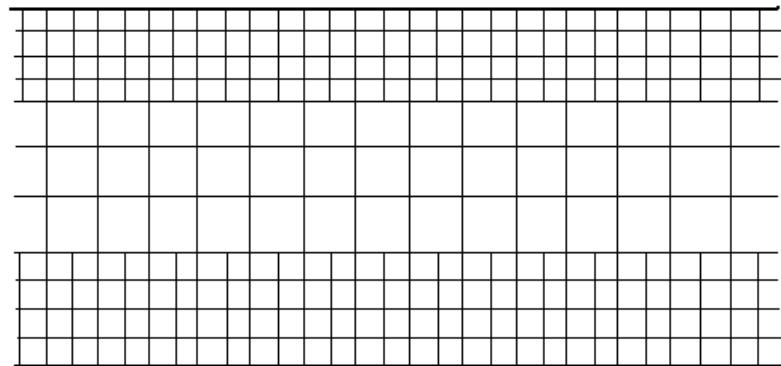
$$\omega = \omega\hat{\mathbf{k}} = \nabla \times \mathbf{u}$$

Vorticity

Non-uniform grids

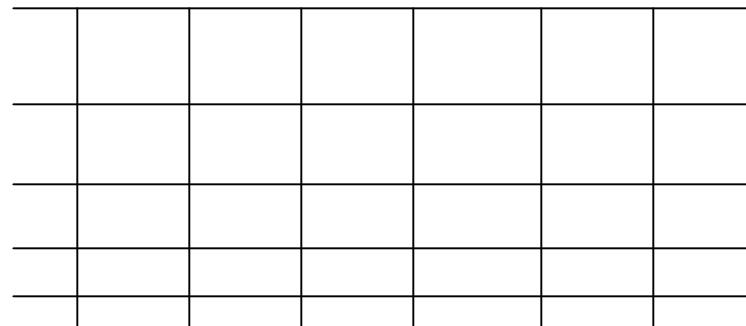
Multiblock

- interpolate on boundaries



Stretched grid

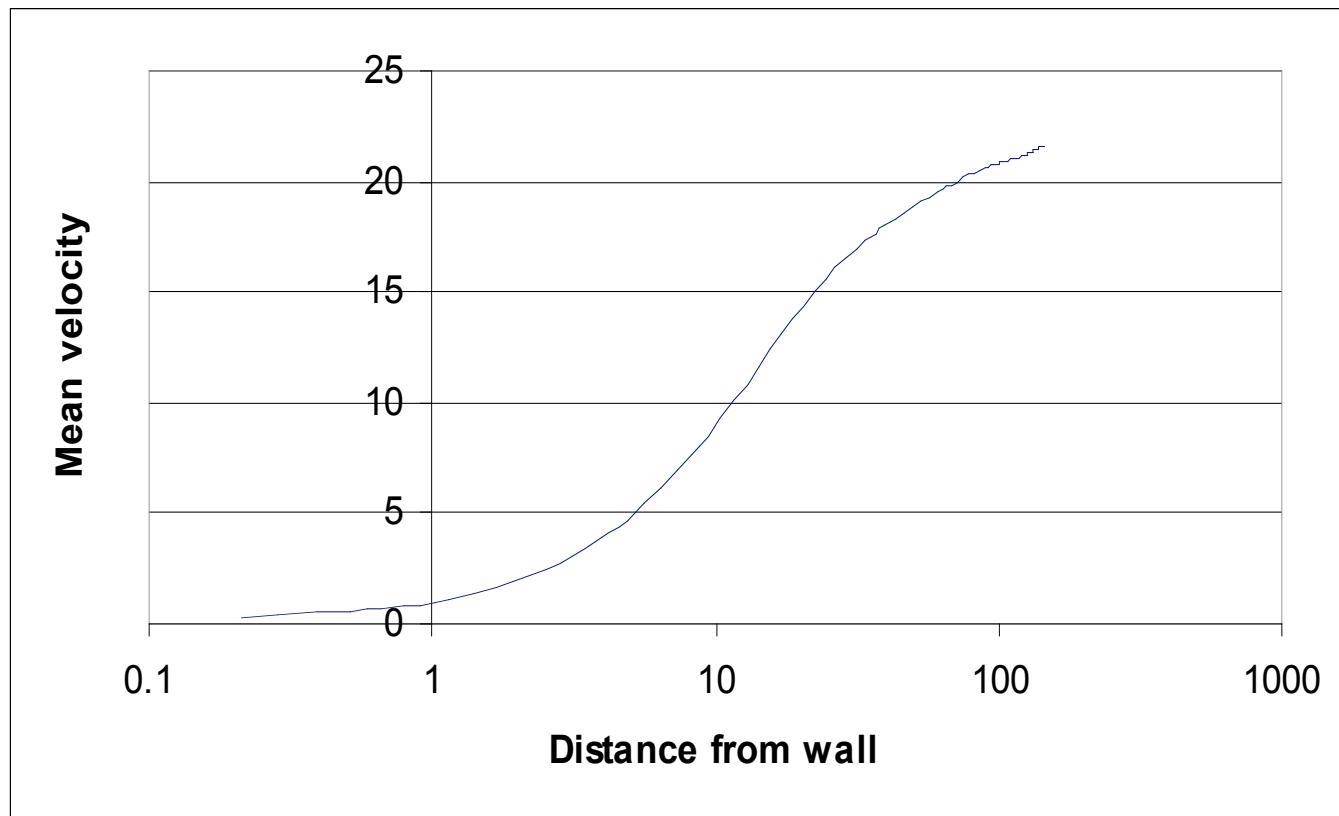
- interpolation before streaming step
- need 2nd order



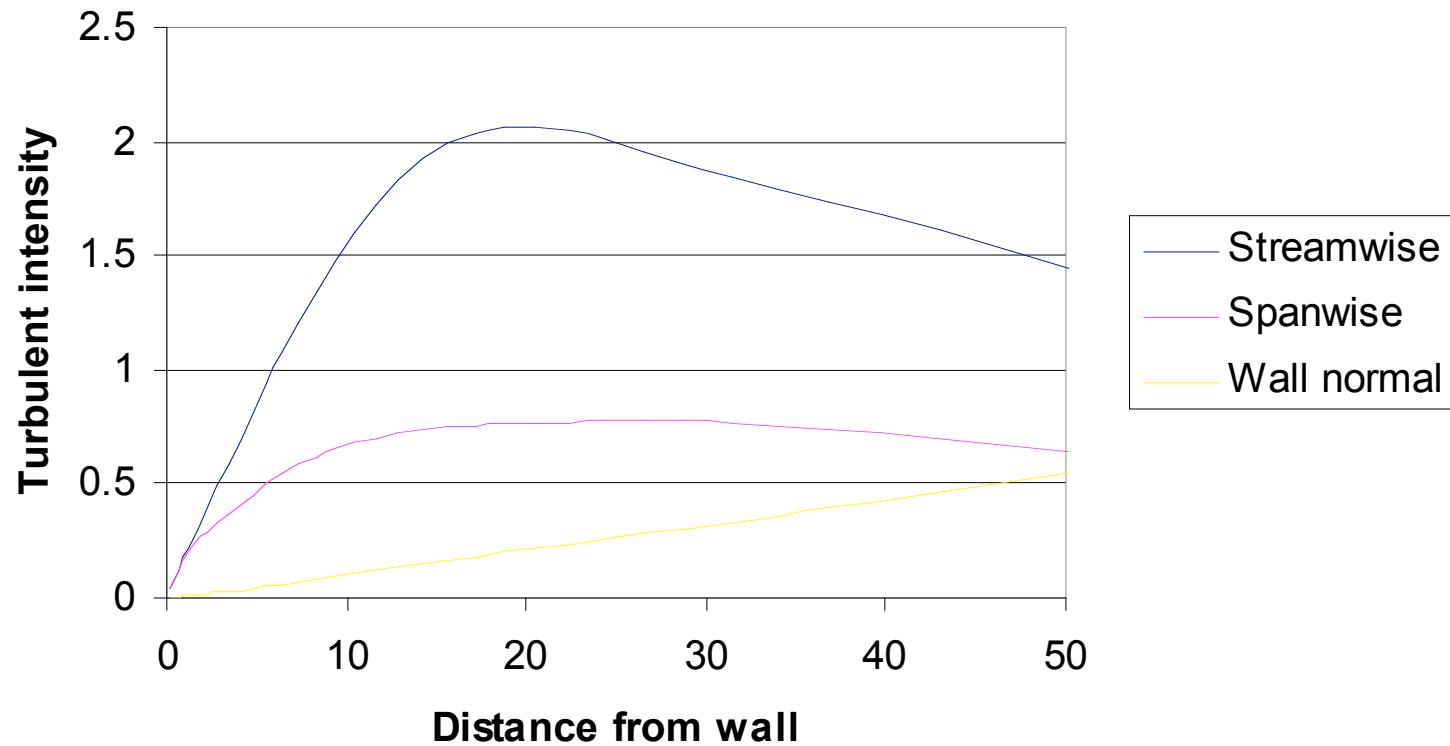
Turbulence Modelling Case

- Free surface flow
- $\text{Re}^* = 171$
- Spectral DNS results available
- Initial run $64 \times 64 \times 80$
- Dimensions $4\pi \times 2\pi \times 2$

Mean Velocity



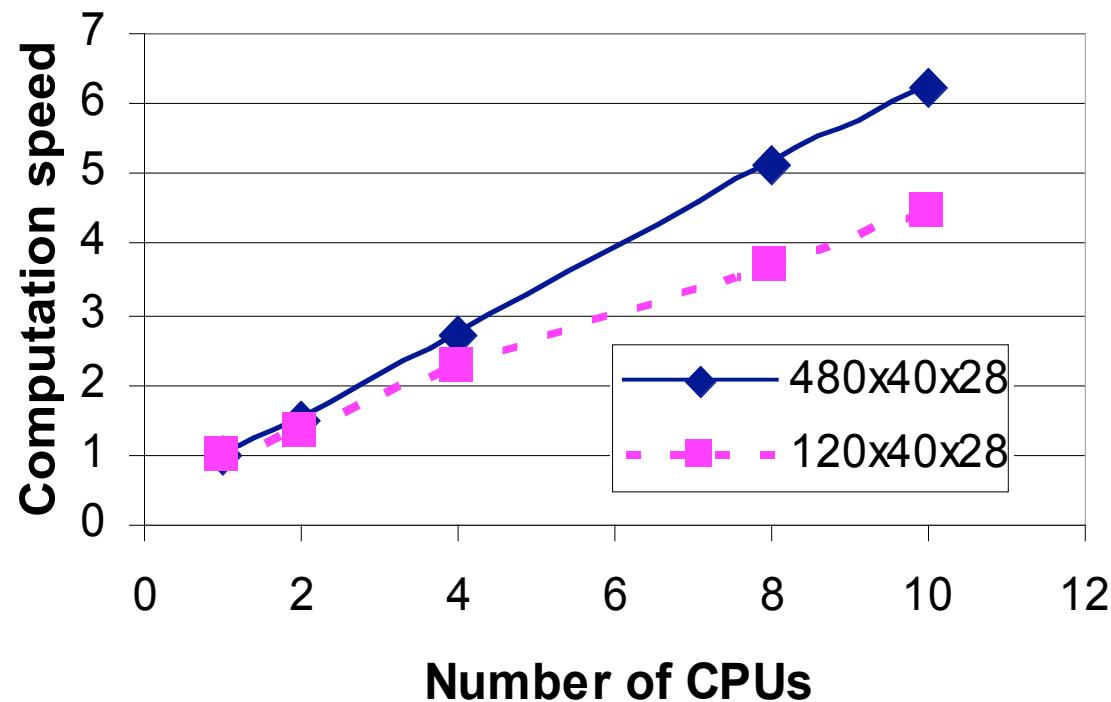
Turbulence Fluctuations



Conclusions & Future Plans

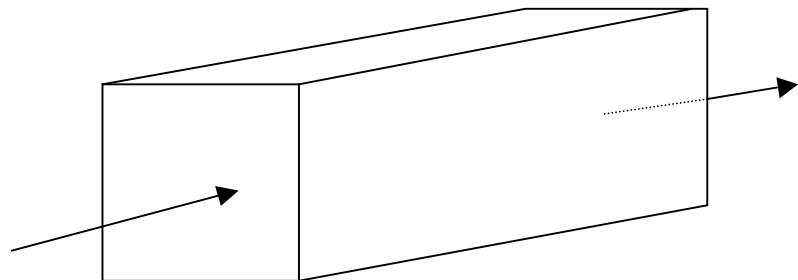
- 2-D MHD code successfully implemented
 - Variable grid implemented
 - Turbulence sustained in expanded grid LB model
-
- Run higher resolution channel flow DNS
 - Modify interpolation scheme
 - Extend MHD model to 3-D
 - Develop BCs for MHD

Parallelisation



Speed of computation for different numbers of CPUs
– plane Poiseuille flow problem

Comparison of Methods



Sample problem:

Flow through rectangular duct
Different grid sizes

For 20 timesteps
same machine
same problem:

